Percentile Methodology for Probability Distributions as Applied to the Representative Scenario Method

Session 81 Statistical techniques for VM-22

Thursday, September 1, 3:15 to 4:45 PM EST

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Percentile Methodology for Probability Distributions as Applied to the Representative Scenario Method

- Notable features of Percentile Methodology
  as Applied to the Representative Scenario Method
- Overview of Representative Scenario Method
- Why Percentile Instead of Standard Deviations
- Overview of Percentile Methodology
- Percentile Methodology Description (8 Steps)
- Percentile Methodology Example

Notable features of Percentile Methodology as Applied to the Representative Scenario Method

- It is simple to implement in contrast to a full-blown stochastic approach
- It is founded on fundamental statistical principles
- It is consistent with the principle based reserving approach
- Parameters of the Key Risk Driver's probability distribution are based on a company’s own experience
- Consistency: Same 8 steps are followed for each Key Risk Driver's probability distribution

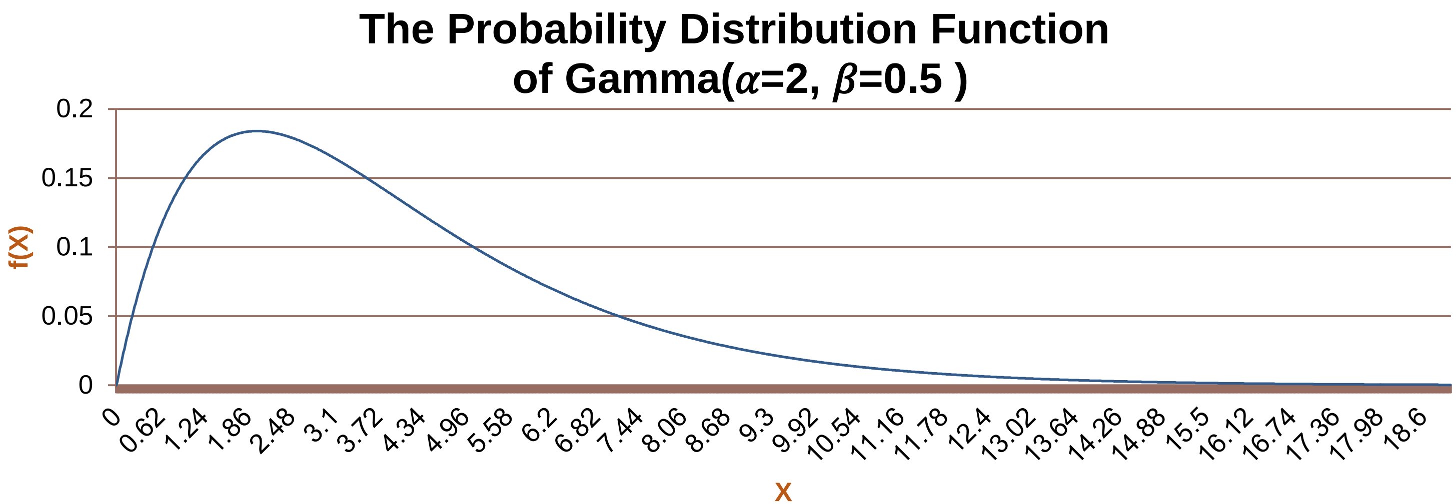
Overview of Representative Scenario Method

- Representative Scenario Method is an auditable modeled reserve reflecting risks of current and emerging products
  - Key Risk Drivers (KRD) vary by features of current and future products
  - Baseline Scenario using the median (50%) value for each KRD
  - Four Scenarios for each KRD
    - 4 Percentiles (Kansas Field Test Used: 99.9%, 84%, 16%,0.1%). Others possible
    - Hold other KRDs at 50%
- Representative Scenario Method has a small number of scenarios that reflect underlying probability distributions, produce results consistent with stochastic methods, and uses aggregate margin

Why Percentiles Instead of Standard Deviations

- Actuaries think in terms of Percentiles
  - Want 2/3 of results (Mean plus or minus 1 Standard Deviation only works for Standard Normal Distribution)
  - CTE 70 – Covers 70% of thousands of stochastic runs
- Problems with Risk Levels and Invalid Results
  - Standard deviations not producing the desired risk level in percentiles.
  - Adding or subtracting a number of Standard Deviations from the Mean can give invalid results.

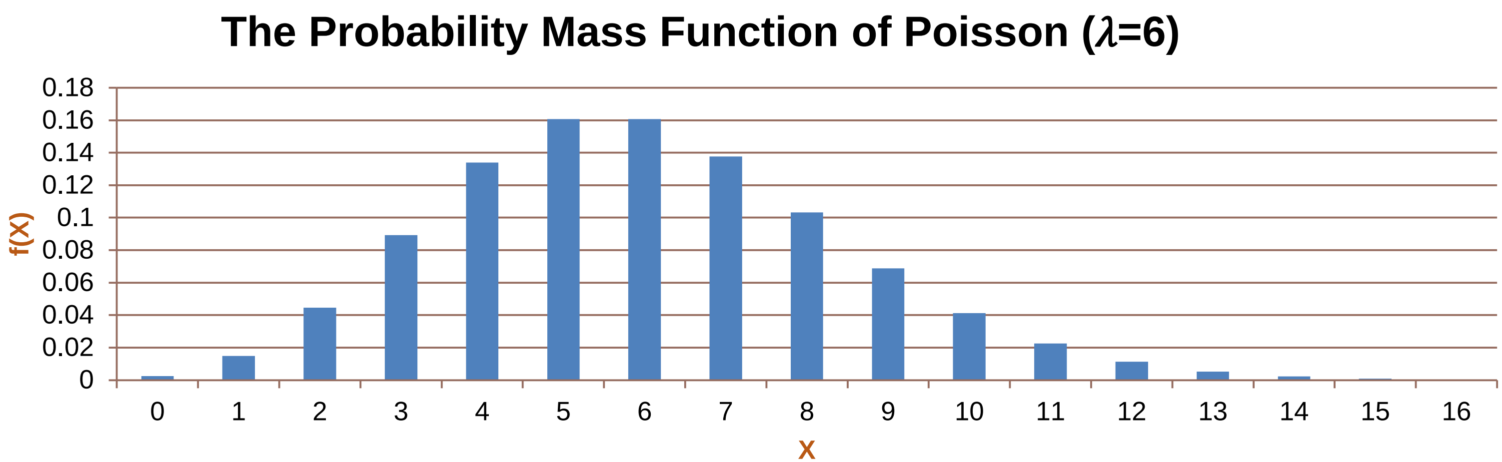
Standard deviations not producing the desired risk level in percentiles.



$$Mean = \frac{\alpha}{\beta} = \frac{2}{0.5} = 4 \text{ and } Standard\ Deviation = \sqrt{\frac{\alpha}{\beta^2}} \approx 2.83$$

	Number of Standard Deviation	Value from Gamma	Risk Level from SD	Anticipated Risk Level
Scenario 1	3 Standard Deviations	12.49	98.6%	99.9%
Scenario 2	1 Standard Deviation	6.83	85.5%	84%
Scenario 3	Mean	4.00	59.4%	50%
Scenario 4	-1 Standard Deviation	1.17	11.7%	16%
Scenario 5	-3 Standard Deviations	-4.49	**Not Defined**	0.1%

Adding or subtracting a number of Standard Deviations from the Mean can give invalid results



$$\text{Mean} = \lambda = 6 \text{ and Standard Deviation} = \sqrt{\lambda} \approx 2.45$$

	Number of Standard Deviation	Value from Poisson	Risk Level from SD	Anticipated Risk Level
Scenario 1	3 Standard Deviations	13	99.6%	99.9%
Scenario 2	1 Standard Deviation	8	84.7%	84%
Scenario 3	Mean	6	60.6%	50%
Scenario 4	-1 Standard Deviation	4	28.5%	16%
Scenario 5	-3 Standard Deviations	-1	Not Defined (Poisson RV must be Non-negative)	0.1%

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Overview of Percentile Methodology

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- Percentile Methodology can use
  - Preset risk levels (e.g. 99.9%, 84%, 16%, 0.1%) or as set by regulator
  - Experience of company
  - Different probability distribution for each KRD
- Percentile Methodology
  - Assigns inputs for each KRD scenario
  - Straight-forward method of weights to calculate KRD reserves

Percentile Methodology Description | 8 Steps

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Same 8 Steps used for different probability distributions

Details can be found in paper at www.mibgroup.com/thoughtleadership

- **Step 1:** Determine the pdf's coefficients based on the company's actual data.
- **Step 2:** Determine Scenario Risk Levels for Each Key Risk Driver.
- **Step 3:** Calculate the Percentile Value for each Key Risk Driver's Risk Level.
- **Step 4:** Calculate Input for the five Scenario Reserves for each Key Risk
- **Step 5:** Determine the Boundaries of the Weight Calculation
- **Step 6:** Determine the Normal CDF of the boundaries for weight calculation range.
- **Step 7:** Calculate the weight of each scenario.
- **Step 8:** Calculate the Key Risk Driver Reserve.

Percentile Methodology Example

For mortality, an appropriate probability distribution is the Poisson Distribution

Poisson Distribution:

- Consistent with events that are rare but significant impact
- Reflects frequency and financial impact of a death claim
- Can be used for meaningful results from low number of deaths within a block of business

- Interesting Fact - In 1837, Siméon-Denis Poisson introduced the distribution that was later used to predict fatalities in the French army due to mule kicks.

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Percentile Methodology Example

Step 1: Determine the pdf's coefficients based on the company's actual data

Assume that, according to an experience study, the number of actual deaths for a company is 20. Therefore, its mortality can be modeled as the Poisson Distribution with Mean=Variance=20 ($\lambda = 20$).

Step 2: Determine Scenario Risk Levels for Each Key Risk Driver

Representative Scenario	Risk Levels
Scenario 1	99.9%
Scenario 2	84%
Scenario 3	50%
Scenario 4	16%
Scenario 5	0.1%

Percentile Methodology Example

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Step 3: Calculate the Percentile Value for each Key Risk Driver's Risk Level.

- Determine the Z Scores of the Standard Normal Distribution at each risk level.
- Estimate the percentile values of the Poisson Distribution using the following formulas and rounding the results to the nearest integer.

$$Upper\ Bound = (A+1) \times \left(1 - \frac{1}{9(A+1)} + \frac{Z}{3\sqrt{(A+1)}}\right)^3$$

$$Lower\ Bound = (A) \times \left(1 - \frac{1}{9A} + \frac{Z}{3\sqrt{A}}\right)^3$$

A: equals to the mean of the Poisson Distribution, Actual deaths. Z: the Z score.

- Upper Bound: the percentiles' risk levels that are above or equal to 50%. For example, 60th percentile, 84th percentile.
- Lower Bound: the percentiles' risk levels that are below 50%. For example, 40th percentile, 16th percentile.

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Percentile Methodology Example

Step 3: Calculate the Percentile Value for each Key Risk Driver's Risk Level

Percentile Values at the Scenario Risk Level are determined and rounded to the nearest integer

Risk Level	Z Score	Percentile Values
99.9%	3.0902	38
84%	0.9945	26*
50%	0	20
16%	-0.9945	16**
0.1%	-3.0902	9

$$* \, 26 = (20+1) \times \left(1 - \frac{1}{9(20+1)} + \frac{0.9945}{3\sqrt{(20+1)}}\right)^3$$

$$** \, 16 = 20 \times \left(1 - \frac{1}{9 \times 20} + \frac{-0.9945}{3\sqrt{20}}\right)^3$$

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Percentile Methodology Example

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Step 4: Calculate Input for the five Scenario Reserves for each Key Risk

The input for the Scenario Generator to calculate the Mortality Key Risk Driver Reserve is done in **three parts**.

- **The first part** is to determine the projected number of deaths at each risk level mortality rate. For the Poisson Distribution this corresponds with the Percentile Values determined in Step 3.

- **The second part** is to determine the company expected deaths based upon a mortality rate from standard mortality table times the company's actual policy exposure. In the third step, the company expected deaths is a constant

Percentile Methodology Example

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Step 4: Calculate Input for the five Scenario Reserves for each Key Risk

The third part is to calculate the input as:

$$Input = \frac{Percentile\ Value}{company\ expected\ deaths}$$

For the 5 Risk Levels

- Assume that for this example the number of company expected deaths is 18 (this number is the product of the company exposure times an industry table). Then the Inputs are:

Risk Level	Percentile Value	Input
99.9%	38	2.111
84%	26	1.444*
50%	20	1.111
16%	16	0.889
0.1%	9	0.500

$$* \; 1.444 = \frac{26}{18}$$

Percentile Methodology Example

Step 5: Determine the Boundaries of the Weight Calculation

The boundaries defining the Weight Calculation Range are the values that are halfway between the Percentile Values calculated in Step 3.

Risk Levels set in Step 2	Half-Way Value	Halfway between the Percentile Values at each risk level
84% and 99.9%	Half-Way Value 1 (H1)	32*
50% and 84%	Half-Way Value 2 (H2)	23
16% and 50%	Half-Way Value 3 (H3)	18
0.1% and 16%	Half-Way Value 4 (H4)	13

$$* \, 32 = \frac{38 + 26}{2} \quad \text{(Rounded to the nearest integer)}$$

Percentile Methodology Example

Step 6: Determine the Poisson CDF of the boundaries for weight calculation range

Risk Levels set in Step 2	Half-Way Value	Halfway between the Percentile Values at each risk level	Poisson CDF
84% and 99.9%	H1	32	0.9953
50% and 84%	H2	23	0.7875
16% and 50%	H3	18	0.3814
0.1% and 16%	H4	13	0.0661

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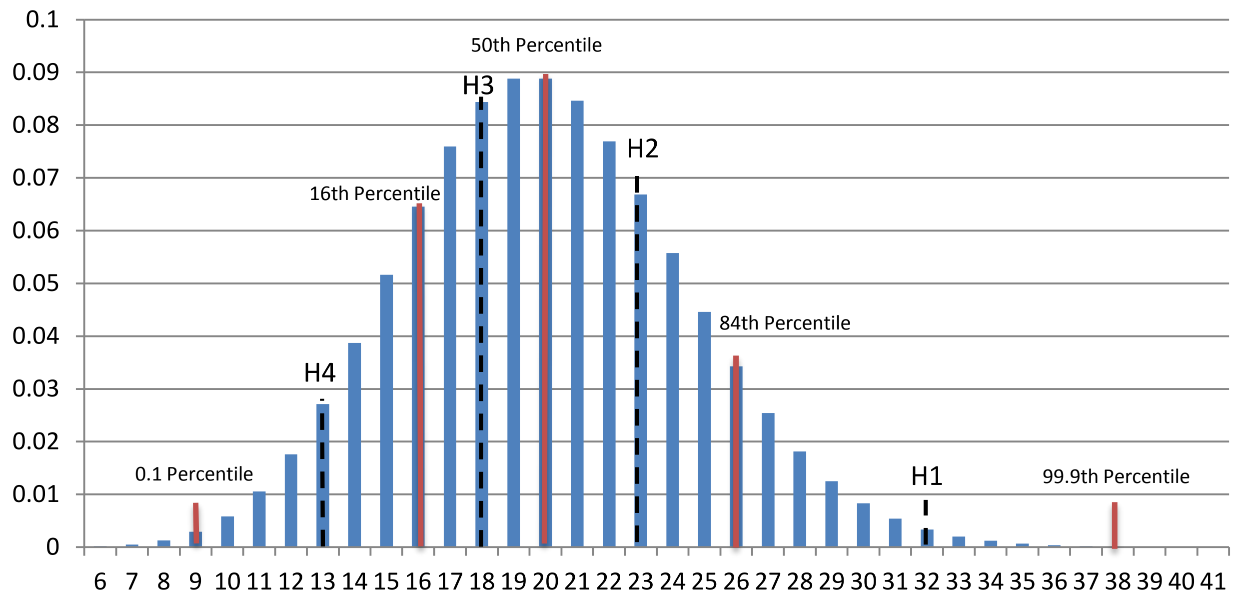
Percentile Methodology Example

Step 7: Calculate the weight of each scenario



Area > H1	Weight for 99.9% Scenario (1- CDF of H1)	0.0047
Area between H1 and H2	Weight for 84% Scenario (CDF of H1 – CDF of H2)	0.2078
Area between H2 and H3	Weight for 50% Scenario (CDF of H2 – CDF of H3)	0.4061
Area between H3 and H4	Weight for 16% Scenario (CDF of H3 – CDF of H4)	0.3153
Area ≤ H4	Weight for 0.1% Scenario (CDF of H4 – 0)	0.0661

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Percentile Methodology Example

Step 8: Calculate the Key Risk Driver Reserve

The Mortality Scenario Reserves for different risk levels are as follows

Risk Level	Mortality Scenario Reserve	Weight
99.9%	800,183,216	0.0047
84%	804,128,122	0.2078
50%	806,084,471	0.4061
16%	808,583,619	0.3153
0.1%	812,611,846	0.0661

$$\text{Mortality Key Risk Driver Reserve}$$
$$= \sum (\text{Mortality Scenario Generator Reserve} \times \text{Unrounded Weight})$$
$$= 806{,}869{,}691$$

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